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The Arrow Semantic Interpreter

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People often sketch diagrams to facilitate their communication. If computers understood the meaning of such diagrams, people could operate information systems more intuitively by sketching. It remains a challenging problem, however, to make computers interpret arrow symbols, which are a frequent and versatile ingredient of such diagrams. This paper develops an algorithm for deducing possible semantic roles of arrow symbols. This, algorithm, called the *Arrow Semantic Interpreter* (*ASI*), emphasizes the elements around arrow symbols and their spatial arrangement, which apparently have a strong influence on the semantics. The semantic roles of arrow symbols are classified into *orientation*, *behavioral description*, *annotation*, and *association*, and for each class the requirements on the arrangement of surrounding elements are identified. By combining these requirements with the rules for adding optional elements, we can deduce potential semantic roles of an arrow symbol from the pattern of surrounding elements. The assessment shows the *ASI* deduces only 1.31 potential semantic roles per arrow symbol, but still they include the correct answer for 79% of sample arrow symbols. This result indicates that the pattern of surrounding elements is highly useful for deriving the interpretation of arrow symbols.

Keywords: Arrow Symbol, Diagram, Interpretation, Sketching Interface, Individual Structure.

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1 Introduction

People often sketch diagrams. Diagrams are often more intuitive than verbal expressions, because we can directly express the spatial and structural characteristics of the information in diagrams. If computers understood such diagrams, people could operate information systems more intuitively, for instance, by sketching diagrams to explain our ideas and knowledge. Indeed, a number of sketch-based computer systems have been developed, and their usefulness has been reported repeatedly (Oviatt 1996; Egenhofer 1997; Landay and Myers 2001; Davis 2002; Ferguson and Forbus 2002). These pioneering systems have demonstrated that computational diagram understanding is a highly promising technology that will enrich human-computer interactions.

Arrow symbols are a frequent ingredient of such diagrams. One reason for the arrow symbols' popularity is their versatility—even though their shapes are extremely simple, they capture a large variety of semantics, such as directions, movements, orders, transitions, and relations. Arrow symbols themselves, however, do not have any specific meaning—they provide the information about the other elements to which the arrow symbols refer. This function of arrow symbols is called their *semantic role* (Kurata and Egenhofer 2005b). Arrow symbols may have a large variety of semantic roles, such as *labeling*, *specifying a direction*, and *illustrating a movement*. Thus, in order to correctly understand a diagram with arrow symbols, its readers have to deduce the semantic roles of each arrow symbol. This is usually not difficult for adults, who have sufficient experience in diagrammatic communication. For sketch-based systems, however, this remains a challenging problem and, accordingly, the current sketch-based systems impose some restrictions on the use of arrow symbols (Section 3). This paper, therefore, develops an algorithm for deducing the semantic roles of arrow symbols. This algorithm is called the *Arrow Semantic Interpreter* (*ASI*). With a capability of deriving interpretations of arrow symbols, sketch-based information systems will allow people to use arrow symbols as naturally as they do in human communication.

The semantic role of an arrow symbol depends on what elements the arrow symbol refers to, and also how the arrow symbol refers to those elements (e.g., attaching oneself to one element or linking two elements) (Kurata and Egenhofer 2005a). Therefore, the *ASI* emphasizes the influence of these elements and their spatial arrangement around the arrow symbol. The combination of arrow symbols with the elements to which the arrow symbols refer is considered a syntactic unit, called an *arrow diagram*, and the elements to which the arrow symbols refer are called the *components* of the arrow diagram (Kurata and Egenhofer 2005a). A component may be represented by an icon, a text label, or a specific position in a map or a background drawing. In order to systematically study the influence of components and their spatial arrangement, this paper develops a model of components' spatial arrangement and their patterns (Section 5).

In addition to the components and their arrangements, visual appearance of arrow symbols, reader's background knowledge, and context may also influence the semantic roles of arrow symbols (Kurata and Egenhofer 2006a; Tversky *et al.* 2007). To simplify the discussion, however, this paper ignores these factors, considering that these factors contribute to the judgment of the validity of interpretations, but not directly to the derivation of the interpretations.

The remainder of this paper is structured as follows: Section 2 reviews the studies on arrow symbols. Section 3 reviews sketch-based systems, highlighting how they deal with arrow symbols. Section 4 distinguishes four classes of arrow symbols' semantic roles. Section 5 develops a model of spatial arrangement of components around arrow symbols, which is called the *individual structure*. Section 6 identifies the requirements on the individual structures when arrow symbols are used for each class of semantic roles. Based on these requirements and the rules for adding optional components (Section 7), we can determine potential semantic roles of arrow symbols from the pattern of components' arrangement. This algorithm is explained in Section 8. Section 9 evaluates the developed algorithm using sample arrow symbols. Finally, Section 10 concludes with a discussion of future problems.

2 Studies on Arrow Symbols

Tversky (2001) defined an arrow symbol as *a special kind of line, with one end marked, inducing an asymmetry*. This definition highlights two essential features of arrow symbols: linearity and asymmetry. With these two features, an arrow symbol establishes an affordance (Gibson 1979) to prompt the diagram readers to move their attention from the tail side to the head side of the arrow symbol. Accordingly, if the arrow symbol connects two elements, these elements are naturally ordered. Also, if the diagram is mapped onto a physical or conceptual space, people naturally imagine a movement in this space. Thanks to these characteristics, arrow symbols have a large variety of semantic roles.

Semantic diversity of arrow symbols has attracted researchers' interest. Van der Waarde and Westendorp (2000) identified seven usages of arrow symbols in pictorial user instructions: moving direction, physical change or transformation, indication of a dimension, labeling, focusing the attention, indication of a sequence, and a part of designed symbols. Similarly, Blaser (2000) identified four usages of arrow symbols in sketch maps: pointing north, indicating a path direction, indicating the direction of a view, and describing where a road leads to. Horn (1998) collected various usages of arrow symbols, which are schematized them in a tree graph. Kurata and Egenhofer (2005b) also collected various semantic roles of arrow symbols, which are classified into four groups: *direction*, *action*, *annotation*, and *conjunction*. In addition, Kurata and Egenhofer (2006b) analyzed the semantic roles assigned to a group of arrow symbols, such as the illustration of meeting, separation, diversion, and confluence. Westendorp (2006) discusses how the usages of arrow symbols has diversified in history, pointing out that the widespread use of pictorial instructions in modern days largely influence the diversification.

Another important characteristic of arrow symbols, which contributes to their popularity, is that the use of arrow symbols enables the illustration of dynamic information in a static diagram. Bertin (1983) claimed that arrow symbols are the most efficient and often the only formula for illustrating a complex movement. Monmonier (1990) insisted that arrow symbols are useful for illustrating spatial diffusion of ideas, migrations of tribes and refugees, advances of armies, and so forth. Tversky *et al*. (2000) confirmed by an experiment that the presence of arrow symbols encourage people to interpret a diagram from its causal and functional aspect.

The polysemy of arrow symbols triggers the problem of disambiguation. Tversky *et al.* (2007) insisted that carefully crafted context can disambiguate meanings of depictive symbols, just as they can disambiguate meanings of words. Kurata and Egenhofer (2005b) demonstrated that possible semantic roles of arrow symbols in simple diagrams can be derived from the pattern of components' arrangement, although they left the development of an interpretation algorithm for general arrow symbols for future work.

3 Arrow Symbols in Current Sketch-Based Systems

Over the last ten years, a variety of sketch-based systems have been developed, aiming at more natural and effective human-computer interaction. For instance, *Spatial-Query-by-Sketch* (Egenhofer 1997; Blaser and Egenhofer 2000) enables its users to query spatial data by drawing a sketch map. *ASSIST* (Alvarado and Davis 2001a; 2001b; Davis 2002) interprets a mechanical sketch and predicts how the illustrated mechanism would behave. Similarly, *SketchIT* (Stahovich 1997; Kurtoglu and Stahovich 2002) interprets mechanical sketches and recreates new designs that realize the same functions. Landay and Myers (2001) developed a computer-aided GUI-design support systems, which interprets a hand-drawn screen layout and generates a prototype program.

While these systems were designed for specific tasks, *GeoRep* (Ferguson and Forbus 2000) was developed as a common platform for sketch-based system. *GeoRep* was applied to *sKEA* (Ferguson and Forbus 2002; Forbus and Usher 2002), which enabled its user to teach the computer his or her knowledge by sketching a diagram.

Some systems combined a sketching interface with a speech interface. Egenhofer (1996) developed the framework of *Sketch-and-Talk in GIS*, in which the user explain a place of interest by sketch and speech in order to query spatial data. *QuickSet* (Cohen *et al.* 1997; Johnston 1998; Cohen *et al.* 2000; Oviatt and Cohen 2000) is a multi-modal system for map-based tasks, which is operated by speech and pen input. Similarly, *nuSketch COA creator* (Forbus *et al.* 2001;

Ferguson and Forbus 2002) is a multi-modal system for a map-based task (military action planning) based on *GeoRep*. Finally, *ASSISTANCE* (Davis 2002) is a multi-modal system that facilitates mechanical designs with sketch and verbal input.

Our question is how these sketch-based systems deal with arrow symbols in sketches. Some systems accept only one or a few semantic roles of allow symbols. For instance, in the Landay and Myers' (2001) computer-aided GUI-design system, arrow symbols are primarily used for specifying button-window relations—which window emerges or gets focus when a button is clicked. In *SketchIT*, arrow symbols specify the movable direction of mechanical components. In *ASSIST*, the user may use one arrow symbol for specifying the gravity direction. In *nuSketch COA creator*, arrow symbols with different semantic roles are distinguished by their different shapes. Such restriction of arrow symbols to few semantic roles works effectively for specific tasks, since the ambiguity of arrow symbols is excluded. As a drawback, the users of these systems are forced to get used to such restriction of arrow symbols.

QuickSet accepts arrow symbols with a variety of semantic roles, such as specifying a direction, illustrating a route, and indicating relations. Furthermore, in *sKEA*, arrow symbols may represent arbitrary binary relations. These systems, however, have room for improvement, because the users have to specify the semantic role of every arrow symbol by speech (in *QuickSet*), text input, or menu selection (in *sKEA*). Such specification processes, which are rarely seen in human communication, reduce the smoothness of sketch-based interactions.

Overall, most sketch-based systems do not allow the natural use of arrow symbols, due to the lack of a human-like ability to deduce the semantic roles of arrow symbols. One exception is *ASSISTANCE*, which automatically distinguishes the arrow symbols representing causality and those representing external force. Unfortunately, such distinction depends on the domain-specific rules, which cannot be applied directly to other sketch-based tasks.

4 Classification of Semantic Roles

Arrow symbols have a large variety of semantic roles. In addition, the semantic roles may be assigned to a group of arrow symbols, as well as individual arrow symbols (Kurata and Egenhofer 2006b). To simplify the discussion, this paper only considers the semantic roles assigned to individual arrow symbols and classifies them into four groups (Kurata and Egenhofer 2005b). Figure 1 summarizes the process of the classification.

First, the semantic roles assigned to individual arrow symbols are dichotomized into those that require only one component and those that require at least two components. In the former group, the arrow symbol is attached to the single component. Naturally, the arrow symbol specifies a directional property of this component. This semantic role is called *orientation*.

The latter group, which requires two or more components, is further dichotomized into *behavioral description* and others, depending on whether the arrow symbol reflects the *transition* of a certain entity or not. This transition includes not only movement (spatial transition), but also continuous existence possibly accompanied by a change (temporal transition). Therefore, illustrations of spatial movement, interaction, duration, and change are all categorized into *behavioral description*.

Arrow symbols with the remaining semantic roles simply connect the components, without implying a transition. Among those arrow symbols, those for labeling are exceptional, because the connected components (usually a label and an icon) refer to the same single subject, while the others associate (two) different subjects. Thus, their semantic roles are distinguished into *annotation* and *association*.

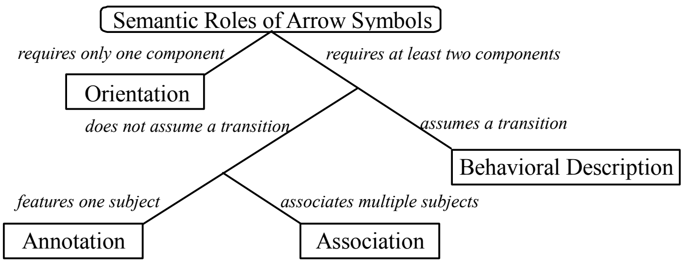


Figure 1. Classification of semantics roles of arrow symbols.

5 Individual Structures of Arrow Symbols

To systematically analyze how the patterns of components' arrangements influence the semantic roles of arrow symbols, this section develops a model of components' arrangements around each arrow symbol. This model is called the *individual structure* associated with the arrow symbol.

When an arrow symbol explicitly refers to a component, this component is located in front of, behind, or along the arrow symbols. This paper, therefore, considers that an arrow symbol is a deictic reference frame (Retz-Schmidt 1988), which identifies three different conceptual areas where the components can be located (Figure 2). These three areas are called the *component slots* of an arrow symbol, or the *tail slot*, *body slot*, and *head slot*, respectively (Kurata and Egenhofer 2005a). Every component, to which an arrow symbol explicitly refer, is assigned uniquely to one of the three component slots, thereby making the distinction of *tail components*, *body components*, and *head components*.

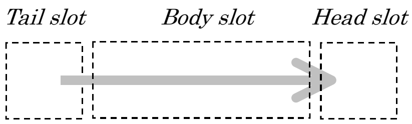


Figure 2. Three component slots associated with an arrow symbol.

The individual structure associated with an arrow symbol *a* (or simply called *a*'s individual structure) is defined as a list of the components in *a*'s three component slots. It is denoted by a 3-tuple, ($C_{tail}(a)$, $C_{body}(a)$, $C_{head}(a)$), where $C_{tail}(a)$, $C_{body}(a)$, and $C_{head}(a)$ are the non-ordered sets of tail, body, and head components, respectively. For instance, the individual structure associated with the arrow symbol in Figure 3 is ({"*Mr. K*", *traveler*}, {"*July 4*"}, {*firework show*, "*Boston*"}).

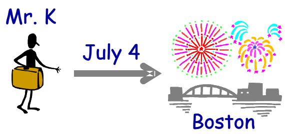


Figure 3. An arrow diagram, which illustrates that a traveler, whose name is Mr. K, goes to a firework show in Boston on July 4th.

The components are dichotomized into *primary components* (*PCs*) and *modifier components* (*MCs*). The primary component represents an independent entity or concept, while the modifier component modifies something else. For instance, in Figure 3, the traveler icon and firework icon are primary components that represent a traveler and a firework show, while the labels "*Mr. K*," "*July 4*," and "*Boston*" are modifier components, which modify the traveler icon, the movement represented by the arrow symbol, and the firework icon, respectively. On the analogy of natural languages, an arrow symbol plays the role of a verb, each primary components corresponds to a subject or an object, and each modifier component correspond to an adjective or an adverb. The distinction of primary and modifier components is purely conceptual. There are, however, the following conventions and rules, which are useful for the distinction of primary and modifier components in a visual domain:

· Icons are usually primary components;
· Text labels attached to icons are usually modifier components
· If an arrow symbol refers to only one component through three slots, this component is a primary component, which is the subject in *orientation* (Section 6); and

- If the head slot has only one component, this component is a primary component, because the modifier component cannot be used in the head slot (Section 7).

The primary components are further categorized into the following four subclasses:

- A *location* (PC_L) is a position in space (e.g., "*Boston*" in Figure 4a). A modifier component may also represent a position in space (e.g., "*Boston*" in Figure 3), but it is not included in the location.
- A *moment* (PC_M) is a position in time (e.g., "*9:00pm*" and "*10:00pm*" in Figure 4b). A modifier component may also represent a position in time (e.g., "*July 4*" in Figure 3), but it is not included in the location.
- An *object* (PC_O) is an entity or its unit, which exists in a physical or conceptual space, and takes an action or gets manipulated (e.g., a vehicle in Figure 4a). Objects are continuants, which endure through some extended interval of time (Worboys and Hornsby 2004).
- An *event* (PC_E) occurs in time, at an instant or over an interval (e.g., a firework show in Figure 4b). It is characterized by a set of changes that the event triggers. Events are occurents, which happen and are then gone (Worboys and Hornsby 2004).

(a) (b)

Figure 4. Two arrow diagrams, which contains (a) a location ("*Boston*") and (b) two moments ("*9:00pm*" and "*10:00pm*").

With the distinction of primary and modifier components (*PC* and *MC*), and further distinction of four subclasses of the primary components (PC_L, PC_M, PC_O, and PC_E), the pattern of the individual structure is defined as follows:

```
pattern_of_individual_structure ::= "(" tail_components
    "," body_components ","head_components ")"
tail_components ::= [components]
body_components ::= [components]
head_components ::= [components]
components      ::= component [components]
component       ::= PC_L|PC_M|PC_O|PC_E|MC
```

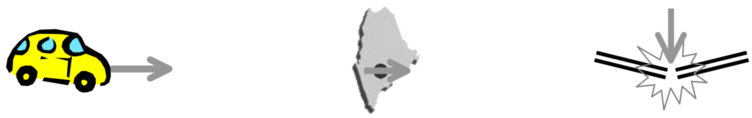
For instance, the individual structures of arrow symbols in Figures 3, 4a, and 4b have the patterns of (*MC* PC_O, *MC*, PC_E *MC*), (PC_O, –, PC_L) and (PC_M, PC_E, PC_M), respectively.

6 Formats of Individual Structures

When using an arrow symbol, we have to arrange the components following certain rules. These rules determine the format of the individual structure associated with the arrow symbol. This section identifies the sets of such formats for the four classes of semantic roles.

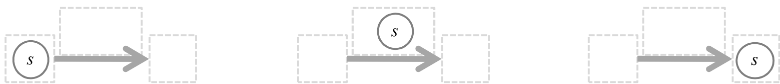
Orientation

An arrow symbol for *orientation* specifies a certain directional property of a component, called the *subject*. The arrow symbol points to, originates from, or passes through/by the subject, typically implying that the directional property is related to an outgoing action, a passing action, or an incoming action, respectively (Figure 5). Accordingly, *orientation* corresponds to the three formats in Figure 6.



(a) (b) (c)

Figure 5. Diagrams with an arrow symbol for *orientation*, specifying (a) a vehicle's moving direction, (b) a wind direction at a geographic location, and (c) the direction of an external force by which a board cracks.



(a) (b) (c)

Figure 6. Formats of individual structures for *orientation* (*s*: subject).

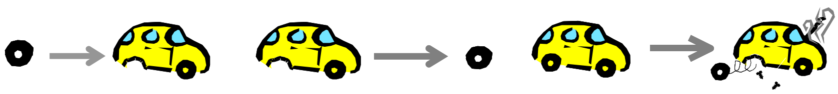
Obviously, the subject must be a primary component, which represents an independent entity or concept. Among the four subclasses of primary components (i.e., location, moment, object, and event), a moment cannot be the subject, since the moment is a zero-dimensional concept and has no directional property. On the other hand, the arrow diagrams in Figure 5, whose subjects are a vehicle, a geographic location, and a board-cracking event, indicate that the subject may be an object, a location, and an event, respectively. Consequently, when an arrow symbol is used for *orientation*, its individual structure satisfies one of the basic patterns in Table 1. In these basic patterns, each blank indicates that the corresponding component slot may be empty or filled by optional components. $PC_{L|O|E}$ represents PC_L, PC_O, or PC_E. For instance, the pattern (PC_L MC, MC, –) satisfies the basic pattern ($PC_{L|O|E}$, ,).

Table 1. Basic patterns of individual structures for *orientation* (s: subject).

Format	Basic patterns
(s, ,)	($PC_{L\|O\|E}$, ,)
(, s,)	(, $PC_{L\|O\|E}$,)
(, , s)	(, , $PC_{L\|O\|E}$)

Behavioral description

An arrow symbol for *behavioral description* illustrates the transition of a component, called the *subject*. In addition to the subject, the arrow symbol may refer to the components, which specify spatial or temporal positions. The tail, body, and head components, if each represents a spatial position, specify the origin, the intermediate point, and the destination of the transition, respectively (Figure 4a). Similarly, the tail, body, and head components, if each represents a temporal position, specify the start time, the intermediate time, and end time of the transition, respectively (Figure 4b). In addition, the arrow symbol may also refer to the components, with which the subject interacts during the transition (Figures 7a-b). These components are called the *participants*. If the participants are located in the tail, body, and head slots, the interaction takes place before, during, and after the transition, respectively.



(a) (b) (c)

Figure 7. Diagrams with an arrow symbol for *behavioral description*, illustrating (a) installation of a wheel, (b) removal of a wheel, (c) change of a vehicle. In (a-c) the subject is the vehicle, and in (a-b) the participant is the wheel.

When an arrow symbol is used for *behavioral description*, its individual structure must satisfy the following constraints:

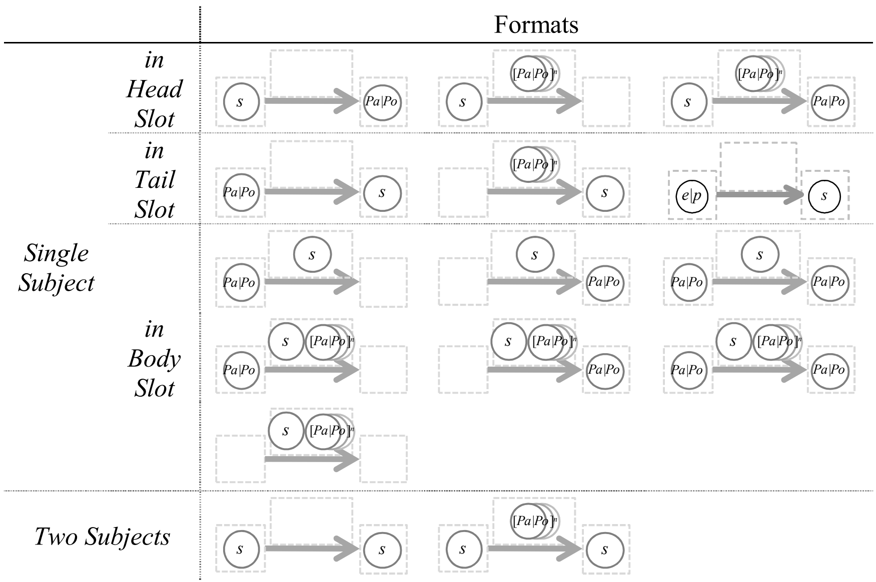
- The subject s is located in any component slot (Figures 7a-b), except when the diagram highlights the change of the subject (Figure 7c).
- In addition to the subject s, the arrow symbol refers to at least one transition-related spatial/temporal position p_o or one participant p_a; otherwise the arrow symbol refers to the subject alone and, accordingly, the semantic role of this arrow symbol becomes *orientation* instead of *behavioral description*.
- The participant p_a cannot be located at the same place with the subject s; otherwise the diagram no longer implies the interaction between s and p_a. In addition, we assume that multiple participants cannot be located at the same place (if apparently multiple participants are located at the same place, they

are considered a single participant as a group). Accordingly, p_a cannot be located in the tail or head slot that already contains s or another p_a.

- The transition-related position p_o cannot be located at the same location with the subject s, the participant p_a, or another transition-related position; otherwise, p_o is regarded an *adjective component* (Section 7). Accordingly, p cannot be located in the tail or head slot that already contains s, p_a, or another p_o.
- The body slot may contain the subject s, one or more participant p_a, and one or more transition-related positions p_a at the same time, because it has length.

These constraints determine the formats of individual structures for *behavioral description* (Table 2).

Table 2. Formats of individual structure for *behavioral description* (s: subject, p_a: participant, p_o: transition-related position). $[X]^n$ represents one or more X.



The subject s, each participant p_a, and transition-related position p_o must be a primary component, as they are independent existence. Among four subclasses of primary components, either an object or an event can be the subject s, since objects and events may change their spatial or temporal positions, but locations and moments cannot. Each participant p_a must be either an object, an event, or a location can be p_a, because they can be a target of action, but a moment cannot. Each transition-related position p_o is obviously either a location or a moment.

Consequently, when an arrow symbol is used for *behavioral description*, its individual structure must satisfy one of the basic patterns in Table 3.

Table 3. Basic patterns of individual structures for *behavioral description* (s: subject, p_a: participant, p_o: transition-related position).

		Format	Basic patterns
Single Subject	*in Head Slot*	$(s,\ ,p_a\vert p_o)$	$(PC_{O\vert E},\ ,PC_{L\vert M\vert O\vert E})$
		$(s,[p_a\vert p_o]^n,\ )$	$(PC_{O\vert E},\ [PC_{L\vert M\vert O\vert E}]^n,\ )$
		$(s,[p_a\vert p_o]^n,p_a\vert p_o)$	$(PC_{O\vert E},\ [PC_{L\vert M\vert O\vert E}]^n,PC_{L\vert M\vert O\vert E})$
	in Tail Slot	$(p_a\vert p_o,\ ,s)$	$(PC_{L\vert M\vert O\vert E},\ ,PC_{O\vert E})$
		$(\ ,[p_a\vert p_o]^n,s)$	$(\ ,\ [PC_{L\vert M\vert O\vert E}]^n,PC_{O\vert E})$
		$(p_a\vert p_o,[p_a\vert p_o]^n,s)$	$(PC_{L\vert M\vert O\vert E}\ [PC_{L\vert M\vert O\vert E}]^n,PC_{O\vert E})$
	in Body Slot	$(p_a\vert p_o,s,\ )$	$(PC_{L\vert M\vert O\vert E},\ PC_{O\vert E},\ )$
		$(\ ,s,p_a\vert p_o)$	$(\ ,PC_{O\vert E},PC_{L\vert M\vert O\vert E})$
		$(p_a\vert p_o,s,p_a\vert p_o)$	$(PC_{L\vert M\vert O\vert E},\ PC_{O\vert E},PC_{L\vert M\vert O\vert E})$
		$(p_a\vert p_o,s[p_a\vert p_o]^n,\ )$	$(PC_{L\vert M\vert O\vert E},\ PC_{O\vert E}[PC_{L\vert M\vert O\vert E}]^n,\ )$
		$(\ ,s[p_a\vert p_o]^n,p_a\vert p_o)$	$(\ ,\ PC_{O\vert E}[PC_{L\vert M\vert O\vert E}]^n,PC_{L\vert M\vert O\vert E})$
		$(p_a\vert p_o,s[p_a\vert p_o]^n,p_a\vert p_o)$	$(PC_{L\vert M\vert O\vert E},\ PC_{O\vert E}[PC_{L\vert M\vert O\vert E}]^n,PC_{L\vert M\vert O\vert E})$
		$(\ ,s[p_a\vert p_o]^n,\ )$	$(\ ,\ PC_{O\vert E}[PC_{L\vert M\vert O\vert E}]^n,\ )$
Two Subjects		$(s,\ ,s)$	$(PC_{O\vert E},\ ,PC_{O\vert E})$
		$(s,[p_a\vert p_o]^n,s)$	$(PC_{O\vert E},\ [PC_{L\vert M\vert O\vert E}]^n,PC_{O\vert E})$

Annotation

An arrow symbol for *annotation* attaches a text component (*label*) to another component (*subject*), thereby specifying the subject's name, category, status, spatial position, temporal position, and so on (Figure 8). Conventionally, *annotation* corresponds to the unique format in Figure 9, where an arrow symbol originates from the label and points to the subject. This format implies that the label is assigned to the subject.

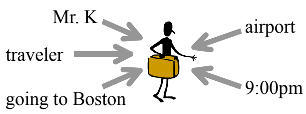


Figure 8. An arrow diagram with five arrow symbols, all used for *annotation*.

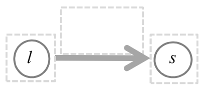


Figure 9. The unique format of individual structures for *annotation* (*l*: label, *s*: subject).

The label, which modifies the subject, is naturally a modifier component, while the subject is a primary component of any subcategory. Consequently, when an arrow symbol is used for *annotation*, its individual structure must satisfy the basic pattern in Table 4.

Table 4. The basic pattern of individual structures for *annotation* (*l*: label, *s*: subject).

Format	Basic pattern
(l, , s)	(MC, , $PC_{L\|M\|O\|E}$)

Association

An arrow symbol for *association* associates two components (*subjects*), indicating the presence of an asymmetric relation between them. Conventionally, these two subjects are placed in the tail and head slots (Figure 10), such that these two subjects look equally emphasized while their order is highlighted.

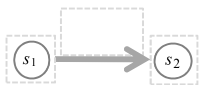


Figure 10. The unique format of individual structures format for *association* (s_1, s_2: associated subjects)

The asymmetric relation, which holds between the two subjects, is called the *effective relation*. The caption, legend, or *adverbial component* (Section 7) may specify the effective relation; otherwise, the diagram reader has to infer the effective relation from the context or their own knowledge about typical relations between the subjects. The effective relation may provide an ordering rationale, which determines the order of the associated subjects (Table 5).

Table 5. Examples of *association* by arrow symbols, where the effective relations between the subjects may naturally determine the subjects' order.

Example	Associated Subjects	Effective Relation	Ordering Rationale
El Niño→Fish Catch↓	El Niño, Fish Catch ↓	Causality	logical order
Plan→Do→See	Plan, Do, See	work process	temporal order
Niagara Falls→Lake Ontario	Niagara Falls, Lake Ontario	water flow	spatial order (high to low)
Lobster→Maine Maine→Lobster	Lobster, Maine	local product	–

Each subject must be a primary component, which represents an independent entity or concept. Any subcategory of primary components can be the subject, as long as people can identify an effective relation between the subjects. Accordingly, when an arrow symbol is used for *association*, its individual structure must satisfy the basic pattern in Table 6.

Table 6. The basic pattern of individual structures for *association* (s_1, s_2: associated subjects).

Format	Basic patterns
$(s_1, \; , s_2)$	$(PC_{L\|M\|O\|E}, \; , PC_{L\|M\|O\|E})$

7 Optional Components

Individual structures of arrow symbols may have optional components, which enrich the diagram's semantics. Two types of optional components are distinguished: *adjective components* and *adverbial components*. An adjective component describes a character of its nearby component, such as name, spatial position, and temporal position (Figure 11). Meanwhile, an adverbial component provides the following information about the semantic role of the arrow symbol:

· When an arrow symbol is used for *orientation*, its adverbial components describe the type, name, and scale of the illustrated directional property (Table 7a$_1$-a$_3$).

· When an arrow symbol is used for *behavioral description*, its adverbial components describe the type and scale, and when and where the transition takes place (Table 7b$_1$-b$_3$).

- When an arrow symbol is used for *association*, its adverbial components describe the effective relation that associates the components and when and where the relation holds (Table 7c_1-c_3).

When an arrow symbol is used for annotation, the arrow symbol does not refer to adverbial components. On the analogy of natural languages, adjective components and adverbial components correspond to adjective and adverbial phrases, which modify a noun and a verb, respectively.

Naturally, both adjective and adverbial components belong to modifier components. Adjective components are placed in the same slot with the modified components. Adverbial components are placed in the body slot, normally around its center, implying that the adverbial component is assigned to the entire arrow symbol.

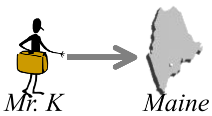
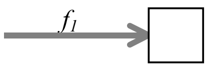
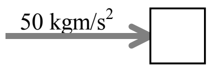
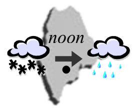
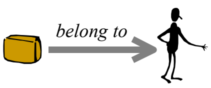


Figure 11. An arrow diagram with two adjective components, "*Mr. K*" and "*Maine*," which specify the name of the nearby components.

Table 7. Information provided by adverbial components.

Semantic Role		Provided Information		
	Orientation	 (a_1) property type	 (a_2) property name	 (a_3) property scale
	Behavioral description	 (b_1) transition type	 (b_2) transition scale	 (b_3) when and where the transition takes place
	Association	 (c_1) effective relation	 (c_2) when and where the relation holds	

8 Deriving Interpretations from Individual Structures

Let us assume that we already know the individual structure of an arrow symbol *a*. By combining the formats of individual structures (Section 6) and the rules of adding optional components (Section 7), we can parse *a*'s individual structure and determine *a*'s all potential semantic roles among *orientation*, *behavioral description*, *annotation*, and *association*.

Let *a*'s individual structure be $s_{\mathrm{ind}}(a) = \left(C_{\mathrm{tail}ij}, C_{\mathrm{body}ij}, C_{\mathrm{head}ij}\right)$, where $C_{\mathrm{tail}}(a)$, $C_{\mathrm{body}}(a)$, and $C_{\mathrm{head}}(a)$ represent the sets of tail, body, and head components, respectively. Let σ_1-σ_4 are *orientation*, *behavioral description*, *annotation*, and *association*, respectively. The arrow symbol *a* nay have a semantic role σ_i only if $s_{\mathrm{ind}}(a)$ satisfies one of the σ_i's formats and every extra component in $s_{\mathrm{ind}}(a)$ is considered as either an adverbial component or an adjective component. Let $\left(\overline{T}_{ij}, \overline{B}_{ij}, \overline{H}_{ij}\right)$ be the j^{th} basic pattern of the semantic role σ_i (Tables 1, 3, 4, and 6). Assume we have a function $ct(c)$ that gives the type of a component *c* among PC_L, PC_M, PC_O, PC_E, and *MC*. With this setting, *a*'s all potential semantic roles are deduced by the following algorithm:

result ← $\{\ \}$
(T, B, H) ← $\left(\{ct(c) \mid c \in C_{\mathrm{tail}}(a)\}, \{ct(c) \mid c \in C_{\mathrm{body}}(a)\}, \{ct(c) \mid c \in C_{\mathrm{head}}(a)\}\right)$
For every semantic role class σ_i
    For σ_i's every basic pattern $\left(\overline{T}_{ij}, \overline{B}_{ij}, \overline{H}_{ij}\right)$
        If $\left(\overline{T}_{ij} \subset T\right) \wedge \left(\overline{B}_{ij} \subset B\right) \wedge \left(\overline{H}_{ij} \subset H\right)$ then
            $\left(T^+, B^+, H^+\right)$ ← $\left(T \setminus \overline{T}_{ij}, B \setminus \overline{B}_{ij}, H \setminus \overline{H}_{ij}\right)$
            $\left(T^*, B^*, H^*\right)$ ← $\left(T^+, B^+, H^+\right)$
            IF $\sigma_i \neq$"Annotation" then remove all *MC* from B^*
            $S = T^* \cup B^* \cup H^*$
            If $\neg\left[\{PC_L \in S\} \vee \{PC_M \in S\} \vee \{PC_O \in S\} \vee \{PC_E \in S\}\right]$ and _
                $\neg\left[\left\{\left(T^* \neq \phi\right) \wedge \left(\overline{T}_{ij} = \phi\right)\right\} \vee \left\{\left(B^* \neq \phi\right) \wedge \left(\overline{B}_{ij} = \phi\right)\right\}\right.$
                $\left.\vee \left\{\left(H^* \neq \phi\right) \wedge \left(\overline{H}_{ij} = \phi\right)\right\}\right]$ _
                then add σ_i to *result*
        End If
    Next
Next

This algorithm is called the *ASI* (*Arrow Semantic Interpreter*). (T, B, H) shows the pattern of *a*'s individual structure $s_{\mathrm{ind}}(a)$. $\left(T^+, B^+, H^+\right)$ shows the extra components in $s_{\mathrm{ind}}(a)$ under the assumption that (T, B, H) satisfies the basic pattern $\left(\overline{T}_{ij}, \overline{B}_{ij}, \overline{H}_{ij}\right)$. If this assumption is correct, every component in $\left(T^{+}, B^{+}, H^{+}\right)$ must be either an adverbial component or an adjective component, and also every component in $\left(T^{*}, B^{*}, H^{*}\right)$ must be an adjective component after line 8. Since each adjective component must be a modifier component, T^{*}, B^{*}, and H^{*} cannot include a primary component PC_L, PC_M, PC_O, or PC_E (line 10). In addition, if T^{*}, B^{*}, and H^{*} contain an adjective component, there must be at least one component in $\overline{T}_{ij}$, $\overline{B}_{ij}$, and $\overline{H}_{ij}$ that the adjective component modifies, respectively (line 10). If these two conditions are satisfied, the semantic role σ_i is recorded as *a*'s potential semantic role. After checking every basic pattern of four classes of semantic roles, the set variable *result* shows *a*'s all potential semantic roles.

9 Evaluation

To evaluate the developed algorithm (i.e., the *ASI*), we developed a prototype program, which deduces all potential semantic roles of an arrow symbol from the pattern of its individual structure (Figure 12). At this stage, the user has to specify the pattern of the individual structure. The automation of this process is a subject for future work.

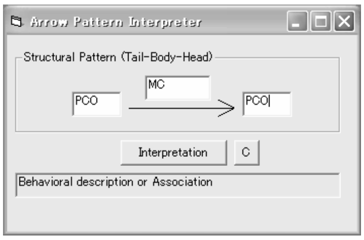


Figure 12. A screenshot of the *ASI*'s prototype.

Sample arrow symbols for the assessment were collected from an introductory GIS textbook, "*Geographical Information Systems and Computer Cartography*" (Jones 1997) because this material satisfies the following conditions:

- the material contains a sufficient number of figures that contain arrow symbols;
- the material is supposed to be read by people without special education or training in diagram reading;
- the figures are neither drawn by few designers nor adhere to in-house aesthetic standards (this is why newspapers and magazines are avoided); and
- the semantic roles of sample arrow symbols are not biased (Figure 14a);

We also examined two more textbooks in biology and astronomy, but the semantic roles of arrow symbols in these two textbooks are considerably biased; the biology textbook predominantly uses arrow symbols for illustrating chemical reactions or organism's movement (both belong to *behavioral description*), while the astronomy textbook frequently uses arrow symbols for illustrating an interval, which cannot be categorized into the four classes of semantic roles that the current *ASI* supports.

The correct semantic roles of sample arrow symbols were assigned manually based on the figures plus context, sometimes drawn from the caption and the body text. We confirmed that the assignment of the correct semantic roles to the arrow symbols correspond to the result of votes by human subjects.

From the GIS textbook, 304 arrow symbols in 64 figures were collected. Some figures contain a large number of similar arrow symbols (Figure 13). These similar arrow symbols, if counted individually, may distort the statistic result. Thus, for every set of similar arrow symbols in each figure (i.e., arrow symbols with the same semantic role and the same pattern of individual structure) one representative is selected. Finally, 94 arrow symbols were prepared for the assessment. This selection process did not bias the proportion of the arrow symbols that belong to the four classes of semantic roles (Figure 14).

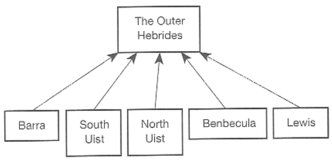


Figure 13. An arrow-containing diagram that contains a large number of similar arrow symbols.

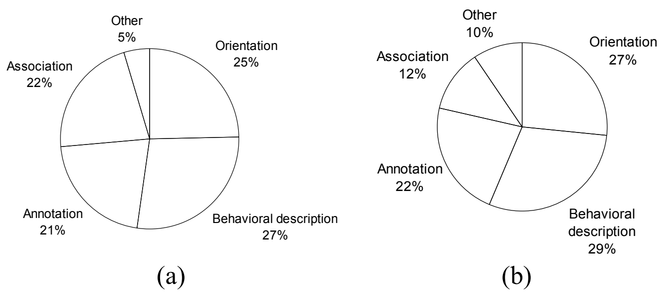


Figure 14. Semantic roles of (a) 304 arrow symbols found in the textbook and (b) 94 arrow symbols selected from the 304 arrow symbols for the assessment.

The evaluation started with counting the number of potential semantic roles that the *ASI* deduced for each arrow symbol. Then, we counted the number of sample arrow symbols whose interpretation yields:

- *exact match*, where the *ASI* deduced only one potential semantic role, which matches the correct semantic role;
- *partial match*, where the *ASI* deduced multiple potential semantic roles, one of which matches the correct semantic role;
- *oversight*, where the potential semantic role(s) deduced by the *ASI* do not include the correct semantic role, even though the correct one is either *orientation*, *behavioral description*, *annotation*, or *association*; and
- *no-answer*, where the potential semantic role (s) deduced by the *ASI* do not include the correct semantic role, simply because the correct one is not among the four classes of semantic roles.

Finally, we calculated the detection rate (i.e., the proportion of sample arrow symbols for which the *ASI* deduced potential semantic role(s) that includes the correct semantic role) and compared it with the detection rate under random choices.

Result

Figure 15 shows the number of potential semantic roles that the *ASI* deduced for the 94 arrow symbols. In most cases the *ASI* deduced one or two potential semantic roles. This result indicates that *ASI* certainly reduces the ambiguity of arrow symbols, since there are initially four choices. For 6% of the sample arrow symbols, however, the *ASI* failed the deduction, due to the use of irregular formats or formats that belong to unsupported semantic roles. On average, 1.31 potential semantic roles were deduced per arrow symbol.

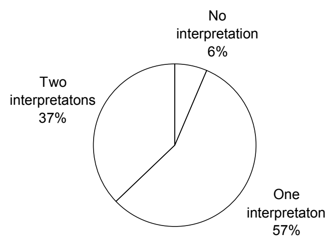


Figure 15. The number of semantic roles that the *ASI* deduced for the 94 arrow symbols.

Figure 16 shows the proportion of the four types of interpretation results. For 44% + 35% = 79% of the sample arrow symbols, the *ASI* successfully detected the correct semantic role. Especially, 44% of the sample arrow symbols yielded an exact match, which requires no further process for narrowing down potential semantic roles. On the other hand, for 11% + 10% = 21% of the sample arrow symbols, the *ASI* failed to detect the correct semantic role. Especially, 10% of the sample arrow symbols were unable to be interpreted because their correct semantic roles were unsupported.

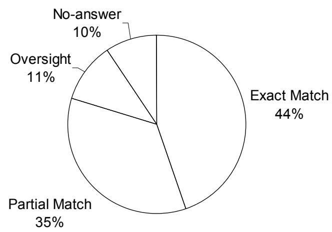


Figure 16. The proportion of the four types of interpretation results for the 94 arrow symbols.

Assume a computer randomly selects zero, one, or two semantic roles from the four classes at the probability of 6%, 57%, and 37%, respectively (Figure 15). Then, the *expected detection rate* (the probability that the randomly-selected potential semantic role(s) include the correct one) is 30% with respect to the 94 sample arrow symbols. On the other hand, the *ASI*'s detection rate is 79%, which is much higher than the expected detection rate under random selection. This result indicates that the *ASI*'s interpretation is highly reliable.

Analysis of Misinterpretations

To identify the reasons for the misinterpretations and the solutions, we analyzed the sample arrow symbols whose interpretation yields *oversight*, *no-answer*, or *partial match*.

Oversight: 11 Cases

In our samples, *oversight* was triggered by the omission of components (Figure 17a) or the use of unsupported formats (Figure 17b). The omission of components occurs when they are obvious from the caption, legend, body text, or context. Accordingly, the use of these information sources is a key for the solution.

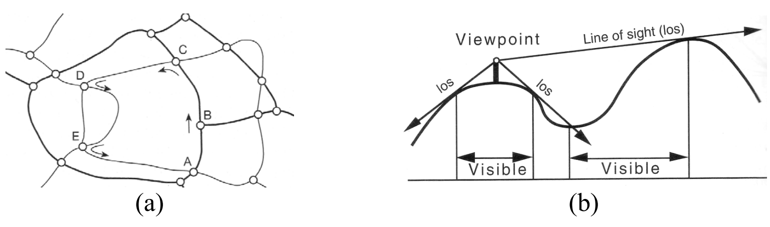


Figure 17. Examples of arrow symbols yielding *oversight*.

No-Answer: 9 cases

Through the analysis of nine *no-answer* cases, the following semantic roles of arrow symbols are newly identified:

· to illustrate an interval (Figure 18a);
· to highlight a certain point in the space (Figure 18b); and
· to imply a gradation (Figure 18c)

The first semantic role is assigned to a pair of arrow symbols, instead of individual arrow symbols. Such group-oriented semantic roles of arrow symbols are related to the formations of the arrow symbols (Kurata and Egenhofer 2006b). It is, therefore, a future problem to develop an interpretation algorithm that considers the formation of arrow symbols in addition to the individual structures. The second semantic role can be regarded as a special case of *annotation* where the label is omitted, because of its obvious message (for instance, *look here*). The third semantic role is probably not as popular as the previous two.

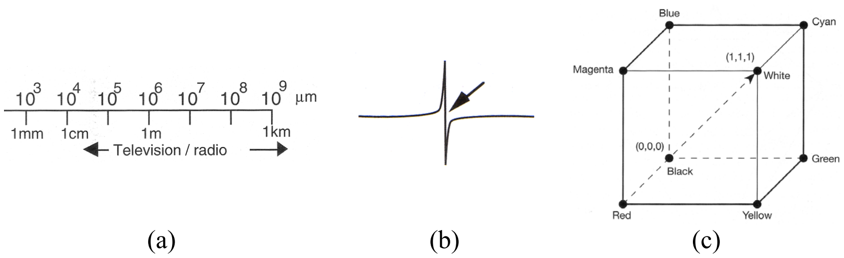


Figure 18. Examples of arrow symbols yielding *no-anwer*. Arrow symbols are used for (a) illustrating an interval, (b) highlighting, and (c) implying a gradation.

Partial Match: 33 Cases

Table 8 summarizes the patterns of the individual structures of 33 arrow symbols that has yielded *partial match*. This table shows that all *partial match* cases has stemmed from the overlap of the patterns that correspond to *behavioral description* and the patterns that correspond to *association*. Among such patterns, ($PC_O[MC]^*$, $[MC]^*$, $PC_O[MC]^*$) is frequently observed (24 out of 33 cases). This pattern is used for three scenarios:

· To illustrate the change of a subject;
· To illustrate an interaction between a subject and a participant; and
· To associate two different subjects.

The first two belong to *behavioral description*, while the last one belongs to *association*. The following clues may be useful for the distinction of these three scenarios:

- If the arrow symbol has non-simple shape, an arrow symbol is usually used for *behavioral description* (in this case, the second scenario), because the arrow symbol's shape often implies the course of spatial transition.
- If two PC_O s cannot be considered two different states of the same subject, an arrow symbol may not illustrate a change.
- If two PC_O are immovable, an arrow symbol may not illustrate an interaction.

Thus, the use of arrow's visual information and background knowledge about components' characteristics becomes an issue. Similar ideas can be applied for the distinction of *behavioral description* and *association* in other cases.

Table 8. Patterns of the individual structures of the 33 arrow symbols which have yielded *partial match*.

Pattern of Individual Structure	Deduced Semantic Roles: Correct	Deduced Semantic Roles: Unnecessary	Num. of Samples
$(PC_O[MC]^*, [MC]^*, PC_O[MC]^*)$	*behavioral description*	*association*	17
$(PC_O[MC]^*, [MC]^*, PC_E[MC]^*)$			3
$(PC_O[MC]^*, [MC]^*, PC_O[MC]^*)$			3
$(PC_E[MC]^*, [MC]^*, PC_L[MC]^*)$			2
$(PC_O[MC]^*, [MC]^*, PC_O[MC]^*)$	*association*	*behavioral description*	7
$(PC_O[MC]^*, [MC]^*, PC_L[MC]^*)$			1

10 Conclusions and Future Work

This paper developed an algorithm for deducing potential semantic roles of arrow symbols. This algorithm makes use of the requirements on components' arrangement, which are specific to each class of semantic roles. The assessment showed that the average number of potential semantic roles deduced by this algorithm is only 1.31, but still they include the correct answer for 79% of sample arrow symbols. This result indicated that the pattern of components' arrangement around each arrow symbol is highly useful information for deriving the interpretation of the arrow symbol. Through the analysis of misinterpreted samples, we identified the following challenges for the improvement:

- Identification of omitted components using captions, legends, and so on;
- Deduction of semantic roles assigned to a group of arrow symbols;
- Judgment of valid semantic role making use of arrow's visual information and background knowledge; and
- Support of additional semantic roles and formats, if necessary.

Another potential challenge is to furnish details to the interpretations. The current algorithm only distinguishes four classes of semantic roles, which might be too coarse for some applications.

Our future goal is to apply the developed algorithm for sketch-based systems in a practical way, such that their users can use arrow symbols freely and naturally in their sketch-based interaction. To achieve this goal, the following techniques are necessary.

- Symbol and text recognition techniques to detect arrow symbols and components in raw sketches.
- Automated linking between arrow symbols and components (i.e., judging whether each arrow symbol refers to each component and, if so, in which slot), probably based on their distance and size.
- Automated distinction of component types. Some diagrammatic conventions help to make the distinction between primary and modifier components (Section 5), but further distinction of their subclasses requires certain background knowledge about the components' characteristics.

The development of these techniques is left for future research.

Arrow symbols are frequently used in people's everyday communication. Especially, they are essential for the communication of spatial and spatio-temporal information. We, therefore, believe that studies on arrow symbols contribute to the improvement of user interfaces in spatial information systems, as well as the development of innovative application systems.

Acknowledgment

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