9^+-Intersection Calculi for Spatial Reasoning on the Topological Relations between Heterogeneous Objects

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ABSTRACT

This paper develops a series of qualitative spatial calculi that feature topological relations based on the 9^+-intersection. While most qualitative spatial calculi have targeted spatial relations between single-type objects, our calculi support spatial relations between heterogeneous pairs of objects. We generalize the rules for deriving compositions of topological relations. As a result, a variety of composition tables can be generated systematically as a foundation of our new calculi. By integrating all relevant sets of topological relations, composition tables, and lists of converse relations, the algebraic framework of ordinary qualitative spatial calculi is successfully reused in our calculi for conducting spatial reasoning. We demonstrate that our 9^+-intersection calculi realize finer reasoning than the former 9-intersection calculi.

Categories and Subject Descriptors

H.2.8 [**Database Management**]: Database Applications – spatial database and GIS

General Terms

Theory

Keywords

topological relations, heterogeneous objects, 9^+-intersection, qualitative spatial reasoning, qualitative spatial calculi

1. INTRODUCTION

Topological relations, which essentially concern how two objects connect or overlap, are one of the most studied sorts of spatial relations in GIS communities. This paper develops a series of qualitative spatial calculi (QSC) that target the topological relations. QSC are the frameworks of qualitative spatial reasoning, with which we can detect inconsistency in the information about possible relations between a set of objects and, if inconsistency is not detected, we can further refine these possible relations. The remarkable feature of our QSC is the ability to conduct spatial reasoning even when the objects are *heterogeneous* (i.e., a mixture of points, lines, and regions). Usually QSC target spatial relations between single-type objects. For instance, Allen's interval algebra [1], Region Connection Calculus [2], Cardinal Direction Calculus [3], and Double Cross Calculus [4] target the relations between two intervals, two regions, two points, and three points, respectively. Such calculi nicely fit into an algebraic framework and thus, spatial reasoning is achieved by algebraic computation (Section 3). In geographic contexts, however, we often deal with the relations between geographic features of different geometric types. Naturally, a question arises as to how we can disambiguate spatial arrangements of such heterogeneous objects. We answer this question with regard to topological relations.

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In GIS communities, both the 9-intersection [5] and Region Connection Calculus [2] have been frequently used as models of topological relations. This work targets the topological relations distinguished by the 9^+-intersection [6, 7], which is a refinement of the 9-intersection. Under the 9^+-intersection, we can semi-automatically identify sets of topological relations, as well as their conceptual neighborhood graphs, for arbitrary pairs of objects [7, 8]. This paper shows that the 9^+-intersection also allows us to generalize the rules for deriving the compositions of topological relations and accordingly, the composition tables, used as a basis of qualitative spatial reasoning, can be derived systematically. This is a great advantage of the 9^+-intersection, because under the 9-intersection the composition tables have been developed for each pair of relation sets in an ad-hoc way. Another advantage is that we can represent and reason on spatial arrangements of objects more precisely than the 9-intersection.

The remainder of this article is structured as follows: Sections 2-3 introduce the 9^+-intersection and basic concepts of QSC, respectively. Section 4 develops the 9^+-intersection calculi. Section 5 demonstrates the use of our new calculi for qualitative spatial reasoning. Finally, Section 6 concludes the discussion. In this article, lines and regions refer to *simple lines* and *simple regions* [9], respectively. We distinguish two terminal points of each line, which we tentatively call the *start point* and *end point*, but this does not mean that the line should have directionality.

2. 9^+-INTERSECTION

Both the 9-intersection [5] and the 9^+-intersection [6, 7] presume the distinction of *topological parts* (*interior*, *exterior*, and *boundary*) of each object. In this paper, the interior, exterior, and boundary of a *n*-dimensional spatial object *X* embedded in a space $\mathcal{S}$, denoted X°, X^-, and ∂X, are defined as the union of all *n*-dimensional open sets contained in *X*, the union of all open sets that do not intersect with *X*, and the difference between the complement of X^- and X°, respectively. By this definition, the boundary of a region and that of a line always refer to the region's bounding edge and the line's two endpoints, respectively, regardless of the dimension of $\mathcal{S}$.

Given two objects A and B embedded in $\mathcal{S}$, the 9-intersection characterizes the topological relation between A and B based on the topological properties of the 3×3 set intersections between the topological parts of A and those of B. These 3×3 set intersections are concisely represented in the *9-intersection matrix* (Equation 1). Usually, topological relations are distinguished simply by the emptiness/non-emptiness of these 3×3 set intersections, which we call the *9-intersection pattern*.

$$\mathrm{M}(A,B)=\begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix} \quad (1)$$

In the 9-intersection, the distinction of the line's two terminal points is lost. This motivated the extension of the 9-intersection by considering the set intersections between the *topological primitives* [6], which are self-connected and mutually-separated subparts of the objects' topological parts. For instance, the boundary of a simple line L consists of two primitives (the start point $\partial_s L$ and the end point $\partial_e L$), while the interior and exterior of L, and the interior, exterior, and boundary of a simple region R, consist of one primitive each. Hence, the topological relation between L and R is characterized by a refined 9-intersection matrix in Equation 2, and the topological relations are distinguished by the emptiness/non-emptiness of the set intersections in this matrix, called the *9⁺-intersection pattern*.

$$\mathrm{M}^+(L,R)=\begin{pmatrix} L^\circ \cap B^\circ & L^\circ \cap \partial R & L^\circ \cap R^- \\ \begin{bmatrix}\partial_s L \cap R^\circ \\ \partial_e L \cap R^\circ\end{bmatrix} & \begin{bmatrix}\partial_s L \cap \partial R \\ \partial_e L \cap \partial R\end{bmatrix} & \begin{bmatrix}\partial_s L \cap R^- \\ \partial_e L \cap R^-\end{bmatrix} \\ L^- \cap R^\circ & L^- \cap \partial R & L^- \cap R^- \end{pmatrix} \quad (2)$$

Given two object domains $\mathbf{D}_\mathrm{A}$ and $\mathbf{D}_\mathrm{B}$ and a space $\mathcal{S}$, the sets of topological relations that may hold between an object in $\mathbf{D}_\mathrm{A}$ and an object in $\mathbf{D}_\mathrm{B}$, distinguished by the 9- and 9⁺-intersection patterns, are denoted $\mathcal{T}_{\mathbf{D}_\mathrm{A}\mathbf{D}_\mathrm{B}\text{-}\mathcal{S}}$ and $\mathcal{T}^+{}_{\mathbf{D}_\mathrm{A}\mathbf{D}_\mathrm{B}\text{-}\mathcal{S}}$, respectively. For instance, $\mathcal{T}^+{}_{\mathbf{LR}\text{-}\mathbb{R}^2}$ is the set of topological line-region relations in $\mathbb{R}^2$ distinguished by the 9⁺-intersection patterns. Note that **P**, **L**, **R**, and **B** represent the domains of points, lines, regions, and bodies. Table 1 lists the numbers of topological relations between all pairs of objects distinguished by the 9- and 9⁺-intersection patterns [7].

Table 1: Numbers of topological relations distinguished by the 9-/9⁺-intersection patterns [7] ($\mathbb{R}^n$: *n*-dimensional Euclidian space, $\mathbb{S}^n$:*n*-sphere, P:point, L:line, R:region, B:body)

Relation	9-intersection					9⁺-intersection				
Type	$\mathbb{R}^1$	$\mathbb{R}^2$	$\mathbb{R}^3$	$\mathbb{S}^1$	$\mathbb{S}^2$	$\mathbb{R}^1$	$\mathbb{R}^2$	$\mathbb{R}^3$	$\mathbb{S}^1$	$\mathbb{S}^2$
P-P	2	2	2	2	2	6	2	2	2	2
P-L/L-P	3	3	3	3	3	10	4	4	4	4
P-R/R-P	–	3	3	–	3	–	3	3	–	3
P-B/B-P	–	–	3	–	–	–	–	3	–	–
L-L	8	33	33	11	33	26	80	80	28	80
L-R/R-L	–	19	31	–	19	–	26	45	–	26
L-B/B-L	–	–	19	–	–	–	–	26	–	–
R-R	–	8	43	–	11	–	8	43	–	11
R-B/B-R	–	–	19	–	–	–	–	19	–	–
B-B	–	–	8	–	–	–	–	8	–	–

3. QUALITATIVE SPATIAL CALCULI

QSC have been studied extensively in AI communities [10]. Each QSC features a specific JEPD (jointly-exhaustive and pairwise disjoint) set of qualitative spatial relations. A *binary* QSC, which features the relations between two objects, normally supports *converse* and *weak/strong composition* operations, in addition to ordinary set-theoretic operations. Let r and s be the spatial relation between A and B and that between B and C. The converse of r, denoted $r^{\smile}$, gives the spatial relation between B and A (usually uniquely determined). The weak composition of r and s, denoted $r \diamond s$, gives the set of all spatial relations that may hold between A and C. The weak composition $r \diamond s$ is an upper approximation of the strong composition $r \circ s$ defined in Equation 3.

$$r \circ s = \{(A,C) \in \boldsymbol{\mathcal{B}} \mid \exists B \in \mathbf{D}\ (A,B) \in r \wedge (B,C) \in s\} \quad (3)$$

The relations that each QSC features are called *base relations* and denoted $\boldsymbol{\mathcal{B}}$ as a set. Many QSC also consider the *general relations*, each referring to a subset of $\boldsymbol{\mathcal{B}}$. By introducing such general relations, every state of knowledge about the possible spatial relations between two objects is represented by a single general relation. For instance, if we know that A is north or northeast of B, this state is represented by a general relation {*north*, *northeast*}. The set of all general relations for $\boldsymbol{\mathcal{B}}$ (i.e., the power set of $\boldsymbol{\mathcal{B}}$) is denoted $\boldsymbol{\mathcal{R}_\mathcal{B}}$. The converse and composition operations on $\boldsymbol{\mathcal{R}_\mathcal{B}}$ are defined based on those on $\boldsymbol{\mathcal{B}}$ as Equations 4-5. Naturally, the converse and composition on $\boldsymbol{\mathcal{R}}$ is closed under $\boldsymbol{\mathcal{R}_\mathcal{B}}$.

$$\forall R_i \in \boldsymbol{\mathcal{R}_\mathcal{B}}\ R_i{}^{\smile} = \bigcup\nolimits_{r \in R_i} r^{\smile} \quad (4)$$

$$\forall R_i, R_j \in \boldsymbol{\mathcal{R}_\mathcal{B}}\ R_i ; R_j = \bigcup\nolimits_{r \in R_i\ s \in R_j} r \diamond s \quad (5)$$

The set $\boldsymbol{\mathcal{R}_\mathcal{B}}$, together with its converse and composition operations, gives rise to an algebra. Normally, a binary QSC forms a *non-associative algebra* or its stronger form. Such an algebraic framework allows us to conduct spatial reasoning by computation. Spatial relations that cannot hold between objects are removed in a stepwise manner by enforcing the constraint of *algebraic closure* [10]. There are already some tools for conducting such spatial reasoning on user-defined QSC (e.g., SparQ [11]).

4. 9⁺-INTERSECTION CALCULI

Following the framework of QSC sketched in the previous section, we formulate a series of qualitative spatial calculi that target the 9⁺-intersection-based topological relations. As a foundation, we first develop converse and composition operations on the 9⁺-intersection-based topological relations.

4.1 Converse

By converse, a topological relation in $\mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_2\text{-}\mathcal{S}}$ is mapped to a topological relation in $\mathcal{T}^+{}_{\mathbf{D}_2\mathbf{D}_1\text{-}\mathcal{S}}$. The *converse list* CL-$\mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_2\mathcal{S}}$ shows the mapping from $\mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_2\text{-}\mathcal{S}}$ to $\mathcal{T}^+{}_{\mathbf{D}_2\mathbf{D}_1\text{-}\mathcal{S}}$ by converse operation. The converse lists of the 9⁺-intersection-based topological relations are derived very easily, because for any relation r in $\mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_2\text{-}\mathcal{S}}$, r and $r^{\smile}$ are represented by a pair of transposed 9⁺-intersection patterns.

4.2 Composition

By composition operation, a pair of a topological relation in $\mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_2\text{-}\mathcal{S}}$ and that in $\mathcal{T}^+{}_{\mathbf{D}_2\mathbf{D}_3\text{-}\mathcal{S}}$ is mapped to a subset of $\mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_3\text{-}\mathcal{S}}$. The *composition table* CT-$\mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_2\mathbf{D}_3\text{-}\mathcal{S}}$ shows the mapping from

$\mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_2\text{-}\mathcal{S}} \times \mathcal{T}^+{}_{\mathbf{D}_2\mathbf{D}_3\text{-}\mathcal{S}}$ to the power-set of $\mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_3\mathcal{S}}$ by composition operation. The composition table is derived as follows: first, for each pair of $r_{AB} \in \mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_2\text{-}\mathcal{S}}$ and $r_{BC} \in \mathcal{T}^+{}_{\mathbf{D}_2\mathbf{D}_3\mathcal{S}}$, we prepare all tuples of (r_{AB}, r_{BC}, r_{AC}) where $r_{AC} \in \mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_3\text{-}\mathcal{S}}$. Then, we remove the tuples that do not satisfy some geometric constraints. After this filtering process, r_{AC} in each remaining tuple becomes a candidate for an element of the composition of r_{AB} and r_{BC}. Each candidate is approved if we can find an instance of the composition by sketching. By repeating this process for every pair of r_{AB} and r_{BC} , we can obtain the composition table CT-$\mathcal{T}^+{}_{\mathbf{D}_1\mathbf{D}_2\mathbf{D}_3\text{-}\mathcal{S}}$.

For the filtering process, we use the following four geometric constraints. Suppose that three objects *A*, *B*, and *C* ($A\in\mathbf{D}_1$, $B\in\mathbf{D}_2$,$C\in\mathbf{D}_3$) are embedded in the space $\mathcal{S}$. Let *X*, *Y*, and *Z* be arbitrary sets of topological primitives of *A*, *B*, and *C* (in arbitrary order). Then, we have the following two constraints:

Constraint 1: If *X* contains *Y* and *Y* intersects with *Z*, then *X* and *Z* intersect.

(Proof) Let $p \in Y \cap Z$. Naturally, $p \in Y$ and $p \in Z$. Since $Y \subseteq X$, $p \in X$. Thus, $p \in X \cap Z$, which indicates that $X \cap Z \neq \emptyset$.

Constraint 2: If *X* contains *Y* and *X* does not intersect with *Z*, then *Y* and *Z* do not intersect.

(Proof) Let $p \in Y$. Since $Y \subseteq X$, $p \in X$. Meanwhile, since $X \cap Z \neq \emptyset$, $p \in X$ implies $p \notin Z$. Therefore, $Y \cap Z = \emptyset$.

Similarly, let *x*, *y*, and *z* be arbitrary topological primitives of *A*, *B*, and *C* (in arbitrary order). Then, we have the following two constraints:

Constraint 3: If *x* contains both *y* and *z*, *x* and *y* are of the same dimensionality, and *z* partially overlaps with *y*, then *z* also intersects with at least one of the adjacent lower-dimensional primitives of *y* (Figure 1a).

(Proof) Since *x* contains both *y* and *z*, *x* can be regarded as a sub-space of $\mathcal{S}$ that embeds *y* and *z*. If $y \subset x$, *y* is bounded on the space *x* by the adjacent lower-dimensional primitive(s) of *y*, since *x* and *y* are of the same dimension. Thus, if *z* overlaps *y*, *z* must intersect with the *bound* of *y* formed by the adjacent lower-dimensional primitive(s) of *y*. On the other hand, $y = x$ is not possible, since *z* cannot overlap with *y*.

Constraint 4: If *x* contains *y*, *y* contains *z*, and the adjacent lower-dimensional primitives of *y* and those of *z* do not intersect, then the adjacent lower-dimensional primitives of *x* and those of *z* also do not intersect (Figure 1b)

(Proof) Let $LA(p)$ be the set of adjacent lower-dimensional primitives of a primitive *p*. In general, $z \subseteq y$ implies $LA(z) \subset (y \cup LA(y))$. In this case, $LA(y) \cap LA(z) = \emptyset$. Thus, $LA(z) \subset$ y. Since $y \subseteq x$, $LA(z) \subset$ x. Since $x \cap LA(x) = \emptyset$, $LA(x) \cap LA(z) = \emptyset$.

(a) (b)

Figure 1. Example configurations for Constraints 3-4

4.3 9⁺-Intersection Calculi

9⁺-intersection calculus for a space $\mathcal{S}$, denoted $9^+{}_{\mathcal{S}}$, is a QSC that targets the 9⁺-intersection-based topological relations between arbitrary simple objects embedded in $\mathcal{S}$. Hence, $9^+{}_{\mathbb{R}^d}$ is built on:

- *d*+1 object domains $\{\mathbf{D}_0, \dots, \mathbf{D}_d\}$ where $\mathbf{D}_0 = \mathbf{P}$, $\mathbf{D}_1 = \mathbf{L}$, $\mathbf{D}_2 = \mathbf{R}$, and $\mathbf{D}_3 = \mathbf{B}$;
- $(d+1)^2$ sets of topological relations $\left\{\mathcal{T}^+{}_{\mathbf{D}_i\mathbf{D}_j\mathbb{R}^d}\right\}$;
- $(d+1)^2$ converse lists $\left\{\text{CL-}\mathcal{T}^+{}_{\mathbf{D}_i\mathbf{D}_j\mathbb{R}^d}\right\}$; and
- $(d+1)^3$ composition tables $\left\{\text{CT-}\mathcal{T}^+{}_{\mathbf{D}_i\mathbf{D}_j\mathbf{D}_k\mathbb{R}^d}\right\}$.

where $i, j, k \in \{0, \dots, d\}$. These elements are adapted to the framework of ordinary QSC using a technique proposed in [12]. First, we introduce a *generalized object domain* $\mathbf{D}^*$ and a set of *generalized base relations* $\mathcal{B}^*$ as follows:

- $\mathbf{D}^* = \bigcup_{i\in\{0,\dots,d\}} \mathbf{D}_i$
- $\mathcal{B}^* = \left(\bigcup_{i\in\{0,\dots,d\}} \mathcal{T}^+{}_{\mathbf{D}_i\mathbf{D}_j\mathbb{R}^d}\right) \cup \{eq^*\}$

$\mathcal{B}^*$ refers to all topological relations between two arbitrary objects in $\mathbf{D}^*$. $\mathcal{B}^*$ also contains a *global identity relation eq**. The presence of an identity relation is a requirement of the algebraic framework (non-associative algebra) of QSC. At the same time, we decided to keep domain-level identity relations ($eq_{\mathbf{PP}}$, $eq_{\mathbf{LL}}$, …), because the reasoning power is reduced if they are replaced by the global identity relation. For instance, the composition of any point-point relation and any line-line relation is impossible and accordingly, $eq_{\mathbf{PP}}; X_{\mathbf{LL}} = \emptyset$ where $X_{\mathbf{LL}}$ is an arbitrary line-line relation. This knowledge is lost if $eq_{\mathbf{PP}}$ is replaced by eq^*, since $eq^*; X_{\mathbf{LL}} = X_{\mathbf{LL}}$. Theoretically speaking, the relations in $\mathcal{B}^*$ must be pairwise disjoint over $\mathbf{D}^* \times \mathbf{D}^*$. To hold this property, eq^* must be *virtual*, in the sense that it satisfies $eq^*; X = X; eq^* = X$, but has no geometric interpretation; in other words, we always use domain-level identity elements for representing equality of two objects.

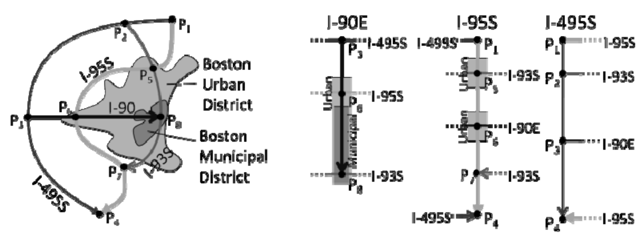
Next, the converse list and composition table for $\mathcal{B}^*$ are prepared. The converse list CL-$\mathcal{T}^+{}_{\mathbf{D}^*\mathbf{D}^*\text{-}\mathbb{R}^d}$ is derived simply by concatenating all relevant converse lists $\left\{\text{CL-}\mathcal{T}^+{}_{\mathbf{D}_i\mathbf{D}_j\text{-}\mathbb{R}^d}\right\}$ and adding an item that indicates $eq^{*\smile} = eq^*$ (See [12]). Similarly, the composition table CT-$\mathcal{T}^+{}_{\mathbf{D}^*\mathbf{D}^*\mathbf{D}^*\text{-}\mathbb{R}^d}$ is derived simply by adjoining all relevant composition tables $\left\{\text{CT-}\mathcal{T}^+{}_{\mathbf{D}_i\mathbf{D}_j\mathbf{D}_k\text{-}\mathbb{R}^d}\right\}$ and adding a row that indicates $eq^* \diamond r = r$ and a column that indicates $r \diamond eq^* = r$ for any relation *r* in $\mathcal{T}^+{}_{\mathbf{D}^*\mathbf{D}^*\text{-}\mathbb{R}^d}$ [12].

Now we have a set of base relations $\mathcal{B}^*$ and converse and composition operations on $\mathcal{B}^*$. These elements satisfy the framework of ordinary qualitative spatial calculi. Consequently, constraint-based spatial reasoning on topological relations between any simple objects can be conducted by algebraic computation.

5. DEMONSTRATION

This section demonstrates how the 9⁺-intersection calculi work in spatial problem solving. We consider the highway network in the Boston metropolitan area (Figure 2a). Imagine that we drive three of the four highways in this area and infer the connection between the remaining highway and Boston's two districts. For instance, when we have driven I-90E, I-95S, and I-495S and obtained the knowledge illustrated in Figure 2b, we can expect that I-93S goes through the two districts. To solve the problems, we use $9^{+}_{\mathbb{R}^2}$, since we have to deal with highway-highway, highway-district, and highway-junction connections, which are represented by topological line-line, line-region, and line-point relations in $\mathbb{R}^2$, respectively. We also use the information about district-district relations (Boston urban district covers Boston municipal district), district-junction relations (which are observable from highways), and junction-junction relation (*disjoint*). For computation, we first register the data of $9^{+}_{\mathbb{R}^2}$ to SparQ [11]. Then, for each drive scenario, we input all topological relations in Figure 12a, except the relations between the unvisited highway we have assumed and the two districts, and compute all consistent scenarios in SparQ.

The derived consistent scenarios show the candidates for the pair of topological relations between the unvisited highway and the two districts. These candidates successfully contained the actual pair of relations. For instance, when we assumed the drive on I-90E, I-95S, and I-495S (Figure 2b), we obtained only one correct solution in which I-93S goes through the two districts. Table 2 shows the numbers of the derived candidates, in comparison with those derived by the corresponding 9-intersection calculus (which we call $9_{\mathbb{R}^2}$) [12]. When we assumed the drive except I-93S, $9^{+}_{\mathbb{R}^2}$ gave a unique result, while $9_{\mathbb{R}^2}$ did not. As pointed out in [12], the 9-intersection omits the information that each line has exactly two endpoints and accordingly, $9_{\mathbb{R}^2}$ derives invalid candidates which presume that the line has more than two endpoints. This problem is successfully avoided under $9^{+}_{\mathbb{R}^2}$, thanks to the clear distinction of the line's two endpoints. In the other three drive scenarios, the numbers of the candidates derived by $9^{+}_{\mathbb{R}^2}$ and $9_{\mathbb{R}^2}$ are the same. This indicates that the results of $9^{+}_{\mathbb{R}^2}$ are actually more informative, since they capture directional characteristics of highway-district relations (e.g., *entering* or *leaving*).



(a) (b)

Figure 2. (a) Spatial arrangement of highways, junctions, and two districts in Boston metropolitan area and (b) information obtained by the drive on I-90E, I-95S, and I-495S

Table 8. Number of candidates for the topological relations between the unvisited highway and two districts

	Unvisited Highway			
	I-90E	I-93S	I-95S	I-495S
$9^{+}_{\mathbb{R}^2}$	1	1	3	6
$9_{\mathbb{R}^2}$	1	6	3	6

6. Conclusions

In geographic contexts, we sometimes deal with heterogeneous objects whose spatial arrangement is of concern. Naturally, GIS communities have developed a number of spatial relation models that target heterogeneous objects, such as topological line-region relations [6] and generalized cardinal direction relations [13]. How to equip such models with the capability of spatial reasoning has been left as a research challenge. In this paper, one of such models, the 9^+-intersection, was built into the existing framework of QSC. The resulting 9^+-intersection calculi enable us to disambiguate spatial arrangements of heterogeneous objects from a topological viewpoint. The 9^+-intersection calculi realize finer reasoning than the 9-intersection calculi [12] when the target includes lines, or potentially any object whose interior, boundary, or exterior consists of multiple subparts.

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